

Radiative Symmetry Breaking from Flat Potential in various $U(1)'$ models

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We investigate a radiative electroweak gauge symmetry breaking scenario via the Coleman-Weinberg mechanism starting from a completely flat Higgs potential at the Planck scale (“flatland scenario”). In our previous paper, we showed that the flatland scenario is possible only when an inequality $K < 1$ among the coefficients of the β functions is satisfied. In this paper, we calculate the number K in various models with an extra $U(1)$ gauge sector in addition to the SM particles. We also show the renormalization group (RG) behaviors of a couple of the models as examples.

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I. INTRODUCTION

Recently the ATLAS and CMS groups reported a discovery of a new particle like the Higgs boson in the standard model (SM) and the mass is near 125 GeV [1]. This means that within the framework of the SM, the perturbation works up to the Planck scale M_{Pl} . It is well-known, however, that this value of mass causes the so-called stability problem of the SM vacuum. Compared with the vacuum expectation value (VEV) of the Higgs field, $v = 246$ GeV, the Higgs mass is relatively small and thus the Higgs potential encounters instability, if we calculate the renormalization group (RG) improved effective potential with the running Higgs quartic coupling $\lambda_H(\mu)$ up to $\mu = M_{Pl}$, where μ is the renormalization scale [2]. If the instability occurs at $\mu < M_{Pl}$, it may suggest an appearance of new physics below the Planck scale M_{Pl} . On the other hand, if the Higgs potential is stable up to M_{Pl} and vanishes there, an interesting possibility can be indicated that the Higgs is borne at the Planck scale as a scalar field with a flat potential [3–5].

Another problem associated with the Higgs potential is that the Higgs mass receives enormous radiative corrections by, if they exist, heavy particles coupled to the Higgs boson. Related to this large radiative corrections to the Higgs mass squared, the naturalness problem has been vigorously examined. Admittedly, the supersymmetry in the TeV scale resolves the naturalness problem, but the LHC and other experiments have given strong constraints on their masses both directly and indirectly. Similarly, the Technicolor scenario was a beautiful idea, but it has been faced with several difficulties, the large S -parameter, the smallness of the discovered Higgs mass, etc.

Because of these difficulties, alternative solutions to the naturalness problem are widely discussed [6] these days. Suppose that the UV completion theory (which may be beyond the ordinary field theories like the string theory) is connected with the SM sector in a way that the SM has no dimensionful parameters. Then if no large intermediate mass scales exist between the SM and the UV completion theory, large logarithmic corrections violating the multiplicative renormalization of the Higgs mass term are never generated: the SM becomes free from the naturalness problem. Such models based on the above idea are called classically conformal models with no intermediate scales [7–10]. In these models, the classical Lagrangian contains no mass terms and all dimensionful parameters are dynamically generated.

Motivated by the vacuum stability and the naturalness problem, we proposed a model that the electroweak (EW) symmetry is radiatively broken in the infrared (IR) region via the Coleman-Weinberg mechanism (CWM) [11] starting from a flat scalar potential in the ultraviolet (UV) region [10]. A more radical possibility that all the scalar potentials vanish at the UV scale was proposed in Ref. [12] and the RG flows were numerically calculated. (See also the corrigendum to [12].) It is, however, rather nontrivial to construct such a model because various couplings must be finely tuned so that the running scalar quartic coupling vanishes both in the IR and the UV regions. In Ref. [13], we

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gave a general criterion for such a scenario, which we call a “flatland scenario”. We showed that an inequality $K < 1$ must be satisfied among the coefficients of the β functions in order to realize the flatland.

In our previous paper [13], we considered the $B - L$ model [14] and calculated the number K . In this paper, we investigate a wider class of the $U(1)$ extension models classified in Ref. [15]. Although the Higgs doublets are assumed to have no charge of the extra $U(1)$ in Ref. [15], we may assign the extra $U(1)$ charges to the Higgs doublets so as to write the SM Yukawa couplings in terms of the mass-dimension four operators. Because there appears the $Z-Z'$ mixing in this case, the ρ -parameter deviates from unity at the tree level. We give a constraint on the model parameters from it. We also propose the minimal vector-like $U(1)'$ model for the flatland scenario, where the SM fermions do not have the $U(1)'$ charges and the spontaneous symmetry breaking (SSB) in the CW sector is transferred into the electroweak symmetry breaking (EWSB) in the SM sector by vector-like fermions having both charges of the SM and $U(1)'$. This vector-like model can be connected with semi-invisible Z' models widely discussed in the literature [16].

The paper is organized as follows: In Sec. II, we study what kind of the $U(1)$ extension models can satisfy the necessary condition for the flatland scenario. In Sec. III, we discuss the ρ -parameter. In Sec. IV, we show realizations of the flatland scenario. Sec. V is devoted to summary and discussions. The full set of the renormalization group equations (RGEs) are shown in Appendix A and B.

II. $U(1)'$ MODELS AND COLEMAN-WEINBERG MECHANISM

It is well known that the CWM does not work within the SM because of the large top Yukawa coupling. Thus we need to extend the SM by introducing an additional sector in which the dynamical mass generation occurs.

We study several extensions of the $U(1)$ sector in the SM including the right-handed neutrinos ν_R [15]. We also introduce the minimal vector-like $U(1)'$ model.

Let us consider a scenario that the Higgs potential is totally flat at some high energy scale Λ and that the extra $U(1)$ breaking via the CWM is encoded into the EWSB through the quartic scalar mixing term. This flatland scenario can be realized only when a certain necessary condition is satisfied [13]. Below we discuss whether or not the flatland scenario works in the $U(1)$ extension models.

It is convenient to rescale the extra $U(1)$ gauge coupling $g_{1'}$ so as to assign the charge of ν_R to -1 . The Majorana Yukawa coupling is then

$$\mathcal{L}_M = -Y_M^{ij} \overline{\nu_{Ri}^c} \nu_{Rj} \Phi + (\text{h.c.}), \quad (1)$$

where the charge of the SM singlet Higgs Φ is $+2$. The VEV of the extra scalar Φ breaks the $U(1)'$ gauge symmetry and provides the mass of Z' . For simplicity, we may take $Y_M^{ij} = \text{diag}(y_M, \dots, y_M, 0, \dots, 0)$ and $\text{tr}[(Y_M^{ij})^2] = N_\nu y_M^2$, etc., where N_ν stands for the number of the right-handed neutrinos having relevant Majorana couplings. The SM Yukawa couplings are discussed separately in a model-dependent way. The scalar potential for Φ is

$$V = \lambda_\Phi |\Phi|^4 + \dots, \quad (2)$$

where the last (\dots) terms are model-dependent. We assume that the Higgs potential has classical conformality, i.e., all of the scalar mass squared terms are vanishing.

The relevant RGEs for the flatland scenario are written as [13]

$$\beta_{g_{1'}} \equiv \mu \frac{\partial}{\partial \mu} g_{1'} = \frac{a}{16\pi^2} g_{1'}^3, \quad (3)$$

$$\beta_{y_M} \equiv \mu \frac{\partial}{\partial \mu} y_M = \frac{y_M}{16\pi^2} \left[b y_M^2 - c g_{1'}^2 \right], \quad (4)$$

$$\beta_{\lambda_\Phi} \equiv \mu \frac{\partial}{\partial \mu} \lambda_\Phi = \frac{1}{16\pi^2} \left[-d y_M^4 + f g_{1'}^4 + \dots \right], \quad (5)$$

with [17]

$$a = \frac{2}{3} \sum_f Q_f Q_f + \frac{1}{3} \sum_s Q_s Q_s, \quad (6)$$

where Q_f and Q_s are the charges of the extra $U(1)$ for two-component (chiral) fermions and complex scalars, respectively. The last dots in β_{λ_Φ} include λ_Φ^2 , $\lambda_\Phi g_{1'}^2$, etc., which are irrelevant in the following analysis.

	$SU(3)_c$	$SU(2)_W$	$U(1)_Y$	$U(1)_{xq-\tau_R^3}$
q_L	3	2	$\frac{1}{6}$	x
u_R	3	1	$\frac{2}{3}$	$4x - 1$
d_R	3	1	$-\frac{1}{3}$	$-2x + 1$
ℓ_L	1	2	$-\frac{1}{2}$	$-3x$
ν_R	1	1	0	-1
e_R	1	1	-1	$-6x + 1$
H	1	2	$+\frac{1}{2}$	$3x - 1$
Φ	1	1	0	$+2$

TABLE I: Charge assignment for $U(1)_{xq-\tau_R^3}$. The x -charges with $x = 1/3$ and $x = 0$ correspond to the $B - L$ and $U(1)_R$ models, respectively.

In order to realize the flatland scenario, the β function for λ_Φ should satisfy $\beta_{\lambda_\Phi} > 0$ in the IR region and simultaneously $\beta_{\lambda_\Phi} < 0$ in the UV region. (For example, see Fig.1 in Ref. [13].) Owing to the IR fixed point of the RGEs [18], we can encode the above inequalities of β_{λ_Φ} into the following necessary condition [13],

$$K \equiv \frac{a+c}{b} \sqrt{\frac{d}{f}} < 1, \quad (7)$$

which is written only in terms of the coefficients of the β functions. Unless the inequality is satisfied, without any concrete calculations for the RG flows, we can conclude that the CWM does not work starting from the flat potential at the UV scale. In this sense, the necessary condition $K < 1$ is a powerful tool for the model-building. Because we rescaled the extra $U(1)$ charge to impose $Q_{\nu_R} = -1$, we obtain b, c, d, f , model-independently,

$$b = 4 + 2N_\nu, \quad c = 6, \quad d = 16N_\nu, \quad f = 96, \quad (8)$$

where we assumed that only the right-handed neutrinos are coupled with the SM singlet scalar Φ . Even if some other fermions interact with Φ , we obtain similar results depending on the form of the interactions. The remaining labor is now only to calculate the coefficient a of $\beta_{g_{1'}}$ in each $U(1)$ extension.

In the following subsections, we will study whether or not the necessary condition $K < 1$ can be satisfied in several chiral $U(1)$ extensions [15]. We also introduce the minimal vector-like $U(1)'$ model with $K < 1$.

A. $U(1)_{xq-\tau_R^3}$

Let us consider the $U(1)$ extension shown in Table I. In Ref. [15], this is represented by $U(1)_{q+xu}$. Notice that we rescaled the $U(1)$ charges so as to impose the charge of ν_R to -1 , as in the $B - L$ model. We may call it $U(1)_{xq-\tau_R^3}$, whose charges correspond to $6xY - \tau_R^3$. Taking $x = 0$ or $x = 1/3$, this model corresponds to $U(1)_R$ or $U(1)_{B-L}$ model, respectively.

The Higgs doublet H does not have the extra $U(1)$ charge in Ref. [15], in order to avoid the ρ -parameter constraint. In this case, there is no contribution to the coefficient a of $\beta_{g_{1'}}$ from H , while the SM Yukawa coupling is not the conventional one. Instead, we may assign the extra $U(1)$ charge to H so as to get the conventional Yukawa couplings¹. One of the cost is that the ρ -parameter deviates from unity at the tree level. We will discuss it later.

The SM Yukawa couplings are then

$$-\mathcal{L}_y = y_u \bar{q}_L u_R \tilde{H} + y_d \bar{q}_L d_R H + y_\ell \bar{\ell}_L e_R H + y_\nu \bar{\ell}_L \nu_R \tilde{H}, \quad (9)$$

where $\tilde{H} \equiv i\tau_2 H^*$. The coefficient a of $\beta_{g_{1'}}$ is

$$a = \left(80x^2 - 32x + \frac{16}{3}\right) N_g + \frac{4}{3} N_\Phi + \frac{2}{3} (3x - 1)^2 N_H, \quad (10)$$

¹ This is essentially the same as the model discussed in the corrigendum to [12].

	$SU(3)_c$	$SU(2)_W$	$U(1)_Y$	$U(1)_{10+x\bar{5}+yY}$
q_L	3	2	$\frac{1}{6}$	$x + y$
u_R	3	1	$\frac{2}{3}$	$-x + 4y$
d_R	3	1	$-\frac{1}{3}$	$1 - 2x - 2y$
ℓ_L	1	2	$-\frac{1}{2}$	$2x - 3y - 1$
ν_R	1	1	0	-1
e_R	1	1	-1	$-x - 6y$
ν'_R	1	1	0	$1 - 5x$
ψ_L^ℓ	1	2	$-\frac{1}{2}$	$1 - 3x - 3y$
ψ_R^ℓ	1	2	$-\frac{1}{2}$	$2x - 3y$
ψ_L^d	3	1	$-\frac{1}{3}$	$-2x - 2y$
ψ_R^d	3	1	$-\frac{1}{3}$	$3x - 2y - 1$
H_U	1	2	$+\frac{1}{2}$	$-2x + 3y$
H_D	1	2	$+\frac{1}{2}$	$3x + 3y - 1$
$H_{\nu'}$	1	2	$+\frac{1}{2}$	$2 - 7x + 3y$
Φ	1	1	0	+2

TABLE II: Charge assignment for $U(1)_{10+x\bar{5}+yY}$.

where N_g represents the number of generations² which couple to the extra $U(1)$ gauge field, $N_\Phi (= 1)$ is the number of Φ , and $N_H (= 1)$ denotes the number of H . We then find

$$K = \frac{(40x^2 - 16x + \frac{8}{3}) N_g + \frac{2}{3}N_\Phi + \frac{1}{3}(3x - 1)^2 N_H + 3}{2 + N_\nu} \sqrt{\frac{N_\nu}{6}}. \quad (11)$$

We show the full set of the RGEs in Appendix A.

For the $B - L$ extension, i.e., $x = 1/3$, we obtain[13]

$$K = \frac{\frac{16}{9}N_g + \frac{2}{3}N_\Phi + 3}{2 + N_\nu} \sqrt{\frac{N_\nu}{6}}. \quad (12)$$

For $N_g = 1, 2$ and $N_\nu = 1$, we find $K = 0.74, 0.98$, respectively, and otherwise, $K > 1$ [13]. Unfortunately, the familiar $B - L$ model with $N_g = 3$ does not work.

How about the general case with an arbitrary x ?

For $N_g = 3$ and $N_\Phi = N_H = 1$, the minimum of a is $a_{\min} = 964/123$, when $x_{\min} = 25/123$. We then find the minimum values of K as follows:

$$K_{\min} = 0.9415, \quad 0.9986, \quad 0.9785 \quad \text{for} \quad N_\nu = 1, 2, 3, \quad (13)$$

respectively. Even for $N_g = 3$, the flatland scenario can work.

We will discuss later the realization of the flatland scenario in this minimal case. Also, the constraint of the ρ -parameter is studied.

B. $U(1)_{10+x\bar{5}+yY}$

Let us consider another extension represented by $U(1)_{10+x\bar{5}}$ in Ref. [15]. The charge assignments are shown in Table II and III. We here added the current of $U(1)_Y$, which does not change the charge of ν_R . We thus call it a $U(1)'_{10+x\bar{5}+yY}$ model. In this extension, we introduce extra vector-like fermions $\psi_{L,R}^\ell$ and $\psi_{L,R}^d$ with respect to the SM charges. Also, there are two kinds of the right-handed neutrinos, which have no SM charges. We always impose

² One may regard them as extra (or “hidden”) generations like in Ref. [19].

	$SU(3)_c$	$SU(2)_W$	$U(1)_Y$	$U(1)'_{10+x\bar{5}+yY}$
q_L	3	2	$\frac{1}{6}$	$x + y$
u_R	3	1	$\frac{2}{3}$	$-x + 4y$
d_R	3	1	$-\frac{1}{3}$	$3x - 2y - 1$
ℓ_L	1	2	$-\frac{1}{2}$	$1 - 3x - 3y$
ν_R	1	1	0	-1
e_R	1	1	-1	$-x - 6y$
ν'_R	1	1	0	$1 - 5x$
ψ_L^ℓ	1	2	$-\frac{1}{2}$	$2x - 3y - 1$
ψ_R^ℓ	1	2	$-\frac{1}{2}$	$2x - 3y$
ψ_L^d	3	1	$-\frac{1}{3}$	$-2x - 2y$
ψ_R^d	3	1	$-\frac{1}{3}$	$1 - 2x - 2y$
H_U	1	2	$+\frac{1}{2}$	$-2x + 3y$
H_D	1	2	$+\frac{1}{2}$	$1 - 2x + 3y$
H_ν	1	2	$+\frac{1}{2}$	$3x + 3y - 2$
Φ	1	1	0	+2

TABLE III: Charge assignment for $U(1)'_{10+x\bar{5}+yY}$.

$Q_{\nu_R} = -1$ and then the other right-handed neutrino has a different $U(1)$ charge. In passing, the charges of d_R and ℓ_L in Table III are just the replacements by those of ψ_R^d and ψ_L^ℓ in Table II, respectively. Depending on them, the charges of the Higgs doublets are determined. In Table II, the SM Yukawa couplings are

$$-\mathcal{L}_y = y_u \bar{q}_L u_R \tilde{H}_U + y_d \bar{q}_L d_R H_D + y_\ell \bar{\ell}_L e_R H_D + y_\nu \bar{\ell}_L \nu_R \tilde{H}_U + y_{\nu'} \bar{\ell}_L \nu'_R \tilde{H}_{\nu'}, \quad (14)$$

while they are

$$-\mathcal{L}_y = y_u \bar{q}_L u_R \tilde{H}_U + y_d \bar{q}_L d_R H_D + y_\ell \bar{\ell}_L e_R H_D + y_\nu \bar{\ell}_L \nu_R \tilde{H}_\nu + y_{\nu'} \bar{\ell}_L \nu'_R \tilde{H}_U, \quad (15)$$

in Table III.

The coefficients a of $\beta_{g_1'}$ are

$$a = (80x^2 - 40x + 120y^2 + 8) N_g + \frac{4}{3} N_\Phi + \frac{2}{3} (2x - 3y)^2 N_{H_U} + \frac{2}{3} (3x + 3y - 1)^2 N_{H_D} + \frac{2}{3} (7x - 3y - 2)^2 N_{H_{\nu'}}, \quad (16)$$

for $U(1)_{10+x\bar{5}+yY}$ in Table II, and

$$a = (80x^2 - 40x + 120y^2 + 8) N_g + \frac{4}{3} N_\Phi + \frac{2}{3} (2x - 3y)^2 N_{H_U} + \frac{2}{3} (2x - 3y - 1)^2 N_{H_D} + \frac{2}{3} (3x + 3y - 2)^2 N_{H_\nu}, \quad (17)$$

for $U(1)'_{10+x\bar{5}+yY}$ in Table III. In both cases, we find

$$a > (80x^2 - 40x + 120y^2 + 8) N_g + \frac{4}{3} N_\Phi, \quad (18)$$

by ignoring the contributions of the Higgs doublets, and thus, for $N_g = 3$ and $N_\Phi = 1$, the lower bound of a is $a > 31/3$ at $x = 1/4$ and $y = 0$. Therefore we obtain

$$K > 1.111, \quad 1.179, \quad 1.155 \quad \text{for} \quad N_\nu = 1, 2, 3, \quad (19)$$

respectively. We conclude that the flatland scenario does not work in the models in Table II and III.

C. $U(1)_{xd-u+yY}$

We next consider a model represented by $U(1)_{d-xu}$ in Ref. [15]. We added the current of $U(1)_Y$ and thus call it a $U(1)_{xd-u+yY}$ model. For the charge assignment, see Table IV. Note that we rescaled the charge so as to get $Q_{\nu_R} = -1$. We corrected the typos of the charge assignments $\psi_{L,R}^{\ell,d}$ in Ref. [15].

	$SU(3)_c$	$SU(2)_W$	$U(1)_Y$	$U(1)_{xd-u+yY}$
q_L	3	2	$\frac{1}{6}$	y
u_R	3	1	$\frac{2}{3}$	$-1 + 4y$
d_R	3	1	$-\frac{1}{3}$	$x - 2y$
ℓ_L	1	2	$-\frac{1}{2}$	$1 - x - 3y$
ν_R	1	1	0	-1
e_R	1	1	-1	$1 - 6y$
ψ_L^ℓ	1	2	$-\frac{1}{2}$	$\frac{1}{5} + x - 3y$
ψ_R^ℓ	1	2	$-\frac{1}{2}$	$\frac{6}{5} - 3y$
ψ_L^d	3	1	$-\frac{1}{3}$	$-\frac{1}{5} - 2y$
ψ_R^d	3	1	$-\frac{1}{3}$	$\frac{4}{5} - x - 2y$
H_U	1	2	$+\frac{1}{2}$	$-1 + 3y$
H_D	1	2	$+\frac{1}{2}$	$-x + 3y$
H_ν	1	2	$+\frac{1}{2}$	$x + 3y - 2$
Φ	1	1	0	+2

TABLE IV: Charge assignment for $U(1)_{xd-u+yY}$.

model	coeff. of $\beta_{g_1'}$	lower bounds of K	
$U(1)_{xq-\tau_R^3}$	$a = (80x^2 - 32x + \frac{16}{3})N_g + \frac{4}{3}N_\Phi + \frac{2}{3}(3x-1)^2N_H$	$N_\nu = 1$	$K \geq 0.9415$
		$N_\nu = 2$	$K \geq 0.9986$
		$N_\nu = 3$	$K \geq 0.9785$
$U(1)_{10+x5+yY}$	$a > (80x^2 - 40x + 120y^2 + 8)N_g + \frac{4}{3}N_\Phi$	$N_\nu = 1$	$K > 1.111$
		$N_\nu = 2$	$K > 1.179$
		$N_\nu = 3$	$K > 1.155$
$U(1)_{xd-u+yY}$	$a = (\frac{20}{3}x^2 - \frac{16}{3}x + 120y^2 - 48y + 8)N_g + \frac{4}{3}N_\Phi + \frac{2}{3}(3y-1)^2N_{H_U} + \frac{2}{3}(x-3y)^2N_{H_D} + \frac{2}{3}(x+3y-2)^2N_{H_\nu}$	$N_\nu = 1$	$K \geq 0.9858$
		$N_\nu = 2$	$K \geq 1.046$
		$N_\nu = 3$	$K \geq 1.024$

TABLE V: Coefficients of the β function of g_1' and lower bounds of K for various $U(1)$ extension models [15]. We fixed to $N_g = 3$ and $N_\Phi = N_H = 1$ (or $N_\Phi = N_{H_U} = N_{H_D} = N_{H_\nu} = 1$). The flatland scenario is possible only when $K < 1$.

From Table IV, the following SM Yukawa couplings are written;

$$-\mathcal{L}_y = y_u \bar{q}_L u_R \tilde{H}_U + y_d \bar{q}_L d_R H_D + y_\ell \bar{\ell}_L e_R H_D + y_\nu \bar{\ell}_L \nu_R \tilde{H}_\nu. \quad (20)$$

The coefficient a of $\beta_{g_1'}$ is

$$a = \left(\frac{20}{3}x^2 - \frac{16}{3}x + 120y^2 - 48y + 8 \right) N_g + \frac{4}{3}N_\Phi + \frac{2}{3}(3y-1)^2N_{H_U} + \frac{2}{3}(x-3y)^2N_{H_D} + \frac{2}{3}(x+3y-2)^2N_{H_\nu}. \quad (21)$$

For $N_g = 3$ and $N_\Phi = N_{H_U} = N_{H_D} = N_{H_\nu} = 1$, the minimum of a is $a_{\min} = 713/84$, when $x = x_{\min} = 7/16$, and $y = y_{\min} = 13/63$. We then find

$$K_{\min} = 0.98579, \quad 1.04559, \quad 1.02446 \quad \text{for} \quad N_\nu = 1, 2, 3, \quad (22)$$

respectively. Even for $N_g = 3$, this model can work, if we take $N_\nu = 1$.

We summarize the results in the subsections A, B, and C in Table V.

D. Minimal vector-like $U(1)'$

In the previous subsections, we studied several chiral $U(1)'$ models. Simpler is a vector-like $U(1)'$ model. Let us consider such a model including the right-handed neutrinos ν_R .

	$SU(3)_c$	$SU(2)_W$	$U(1)_Y$	$U(1)'$	Z_2
ν_R	1	1	0	-1	+1
N_R	1	1	0	+1	-1
ψ_L	1	1	x	y	-1
ψ_R	1	1	x	y	-1
Φ	1	1	0	+2	+1

TABLE VI: Charge assignment for the minimal vector-like $U(1)'$ model. The SM fermions and the Higgs doublet have no $U(1)'$ charge and do even parity of the Z_2 symmetry.

Unlike the previous chiral $U(1)'$ extension, we may divide the model into the SM and CW sectors. In this case, only the CW sector has the $U(1)'$ charge. In order to mediate the SSB of the CW sector to the EWSB in the SM sector, we also need to introduce some fields having both charges of the SM gauge interactions and the extra $U(1)'$. This scenario can be realized by the minimal set of the required matter contents. Also, we note that this setup is similar to the dark Z' models [16].

For the anomaly cancellation from ν_R , we introduce the right-handed fermion N_R with the opposite $U(1)'$ charge to ν_R for each generation. The Majorana Yukawa couplings among Φ , ν_R and N_R are

$$-\mathcal{L}_M = Y_M^{ij} \overline{\nu_{Ri}^c} \Phi \nu_{Rj} + Y_N^{ij} \overline{N_{Ri}} \Phi N_{Rj}^c + (\text{h.c.}). \quad (23)$$

The Majorana mass term $\overline{\nu_R^c} N_R$ is dangerous, because it potentially yields a big correction to the mass term of Φ at the 1-loop level. We thus assign different Z_2 parity to ν_R and N_R in order to forbid this Majorana mass term. As an intermediary between the CW and SM sectors, we also introduce vector-like fermions $\psi_{L,R}$ with both charges of $U(1)_Y$ and $U(1)'$. These fermions causes the nonzero value of the quartic Higgs mixing coupling in low energy through the gauge mixing effects. Although the vector-like mass term for $\psi_{L,R}$ breaks the classical conformality, it does not contribute to the mass terms for H and Φ at the 1-loop level, because there is no interaction among $\psi_{L,R}$, H and Φ . In this sense, the theory is still “natural”. We show the charge assignment in Table VI.

In this model, the coefficient a of $\beta_{g_{1'}}$ is

$$a = \frac{4}{3}N_g + \frac{4}{3}N_\Phi + \frac{4y^2}{3}N_\psi, \quad (24)$$

where N_ψ is the number of $\psi_{L,R}$, and $N_g = 3$. For simplicity, we may take Y_M^{ij} and Y_N^{ij} as diagonal matrices, $Y_{M(N)}^{ij} = \text{diag}(y_{M(N)}, \dots, y_{M(N)}, 0, \dots, 0)$, as was done in the previous subsection, and further simplify them to $y_M = y_N$. In this case,

$$b = 4 + 2N_{\nu_R} + 2N_{N_R}, \quad c = 6, \quad d = 16(N_{\nu_R} + N_{N_R}), \quad f = 96, \quad (25)$$

where N_{ν_R} and N_{N_R} denote the number of ν_R and N_R having the relevant Majorana Yukawa couplings, respectively. The full set of the RGEs are shown in Appendix B.

In this model, we find

$$K = \frac{\frac{2}{3}N_g + \frac{2}{3}N_\Phi + \frac{2y^2}{3}N_\psi + 3}{2 + N_{\nu_R} + N_{N_R}} \sqrt{\frac{N_{\nu_R} + N_{N_R}}{6}}. \quad (26)$$

For simplicity, taking $N_g = 3$, $N_\Phi = 1$, $N_\psi = 1$, $N_{\nu_R} = N_{N_R}$, and $y = 1$, we obtain

$$K = 0.914, 0.862, 0.792, \quad \text{for } N_{\nu_R} = N_{N_R} = 1, 2, 3. \quad (27)$$

In this way, the flatland scenario is easily realized in the minimal vector-like model.

III. ρ -PARAMETER

In the previous section, we introduced (multiple) Higgs doublet(s) with extra $U(1)$ charges. This is, however, very dangerous, because the ρ -parameter deviates from unity at the tree level.

Let us revisit a formula of the ρ -parameter [14].

In general, we introduce multiple Higgs doublets H_k with $k = 1, 2, \dots, N_H$. The hypercharge of H_k is commonly $Y_H = 1/2$ by definition and the extra $U(1)$ charges are Y'_k . The VEVs of H_k are represented by v_k . The Φ for the extra $U(1)$ breaking does not have the hypercharge. The covariant derivative is then

$$D_\mu = \partial_\mu - ig_2 \frac{\tau^i}{2} W_\mu^i - iY(g_Y B_\mu + g_{\text{mix}} B'_\mu) - ig_{1'} Y' B'_\mu, \quad (28)$$

where the gauge mixing g_{mix} appears and Y' denotes the charge of $U(1)'$. After diagonalizing the mass matrix of the neutral gauge boson, we obtain the masses of Z and Z' ($M_Z < M_{Z'}$). The tree level ρ -parameter is defined by $\rho_0 = M_W^2/c_W^2 M_{Z'}^2$, and hence we obtain a simple formula for the deviation of the ρ -parameter from unity as follows:

$$\delta\rho \equiv \rho_0 - 1 = \frac{\tan\theta_{ZZ'} \sum_k (g_{\text{mix}} + 2Y'_k g_{1'}) \frac{v_k^2}{v^2}}{\frac{g_2}{c_W} - \tan\theta_{ZZ'} \sum_k (g_{\text{mix}} + 2Y'_k g_{1'}) \frac{v_k^2}{v^2}}, \quad (29)$$

with the Z - Z' mixing $\theta_{ZZ'}$,

$$\tan\theta_{ZZ'} = \frac{c_W}{g_2} \frac{M_0^2}{M_{Z'}^2 - M_0^2} \sum_k (g_{\text{mix}} + 2Y'_k g_{1'}) \frac{v_k^2}{v^2}, \quad (30)$$

where we used the SM formula of the Z mass, $M_0^2 \equiv g_2^2 v^2 / (4c_W^2)$, and $c_W = \cos\theta_W = e/g_Y$. Since the Z mass M_Z is always smaller than the SM formula M_0 , we generally find $\delta\rho > 0$.

The numerator of Eq. (29) should be small. Otherwise, $\delta\rho$ becomes terribly large. We then obtain approximately,

$$\delta\rho \simeq \frac{v^2}{4M_{Z'}^2} \left(\sum_k (g_{\text{mix}} + 2Y'_k g_{1'}) \frac{v_k^2}{v^2} \right)^2. \quad (31)$$

Compared with the experimental bound $\rho_0 = 1.0004^{+0.0003}_{-0.0004}$ [20], we find

$$\frac{v}{M_{Z'}} \left| \sum_k (g_{\text{mix}} + 2Y'_k g_{1'}) \frac{v_k^2}{v^2} \right| \lesssim 0.05. \quad (32)$$

In terms of $\theta_{ZZ'}$ with $M_{Z'} \sim 1$ TeV, the above inequality corresponds to $|\theta_{ZZ'}| \lesssim 10^{-3}$, which agrees with the conventionally quoted bound [14]. Since the bound by the particle data group [20] includes higher order corrections, one might not take the number of the inequality (32) at face value. At least an order of magnitude of the Z - Z' mixing effect is limited by (32).

In order to construct realistic models, we need to survey other constraints from the precision measurements, the bounds of the direct searches at Tevatron and LHC, etc. [21–23]. These constraints depend on details of the charge assignments of the SM fermions, however. Such model-dependent analyses will be performed elsewhere. Last but not least, it is noticeable that the Z - Z' mixing angle $\theta_{ZZ'}$ is observable at the LHC and/or at the future ILC [24, 25].

IV. REALIZATIONS OF FLATLAND SCENARIO

As concrete realizations of the flatland scenario, we take the chiral $U(1)_{xq-\tau_R^3}$ and vector-like $U(1)'$ models in Sec. II. The potential V for H and Φ is

$$V = \lambda_H |H|^4 + \lambda_\Phi |\Phi|^4 + \lambda_{\text{mix}} |H|^2 |\Phi|^2, \quad (33)$$

where we assumed the classically conformality for the scalar fields.

In the flatland scenario, we impose vanishing of the scalar potential at a UV scale Λ ,

$$\lambda_H(\Lambda) = 0, \quad \lambda_\Phi(\Lambda) = 0, \quad \lambda_{\text{mix}}(\Lambda) = 0. \quad (34)$$

We also set $g_{\text{mix}}(\Lambda) = 0$ by constructing a model with no $U(1)$ kinetic mixing at the high energy scale Λ .

For appropriate parameters investigated below, the CWM occurs in the $U(1)'$ sector of Φ and thereby Φ acquires the VEV, $\langle\Phi\rangle = v_\Phi/\sqrt{2}$, [8, 10]. Then the EWSB takes place if the H - Φ mixing term $\lambda_{\text{mix}} |H|^2 |\Phi|^2$ is negative;

$$v_H^2 = \frac{-\lambda_{\text{mix}}}{2\lambda_H} v_\Phi^2, \quad (35)$$

where $\langle H \rangle = (0, v_H/\sqrt{2})^T$. The Higgs mass m_H is approximately given by $m_H^2 = 2\lambda_H v_H^2$, because the mixing between H and Φ is tiny. In [10], we have shown that such a small and negative scalar mixing is radiatively generated through the gauge kinetic mixing of $U(1)_{B-L}$ and $U(1)_Y$.

Starting from the flat potential (34) at the UV scale, running couplings are obtained numerically by solving the RGEs, and the value of v_H in Eq. (35) is predicted in terms of them. The RG flows are controlled by, besides the SM parameters, the $U(1)'$ gauge coupling and the Majorana Yukawa couplings at Λ . Given these two parameters at Λ , the symmetry breaking scales of Φ and H are determined. In order to set $v_H = 246$ GeV, we must adjust one of the two parameters in accordance with the other. Hence there is only *one free parameter* in the model. In particular, the CW relation

$$\frac{m_\phi^2}{M_{Z'}^2} + N_\nu \frac{g_{1'}^2}{\pi^2} \frac{M_{\nu_R}^4}{M_{Z'}^4} \simeq \frac{3g_{1'}^2}{2\pi^2} \quad (36)$$

must hold, where we used

$$m_\phi^2 \simeq -4\lambda_\Phi v_\Phi^2, \quad M_{Z'} \simeq 2g_{1'} v_\Phi, \quad M_{\nu_R} \simeq \sqrt{2} y_M v_\Phi. \quad (37)$$

Note that the CW relation comes from the relation between the running scalar coupling λ_Φ and the β function β_{λ_Φ} ; $\beta_{\lambda_\Phi} \simeq -4\lambda_\Phi$ at the CW scale $\mu = v_\Phi$ (see e.g., eq.(4) in [8]).

In the following analysis, we take the UV scale at $\Lambda = 1/\sqrt{8\pi G} = 2.435 \times 10^{18}$ GeV. Also, we fix the Higgs mass, $m_h = 126.8$ GeV and the \overline{MS} mass of the top quark³ $\overline{m}_t \simeq 160$ GeV so as to realize $\lambda_H(\Lambda) = 0$.

A. $U(1)_{xq-\tau_R^3}$

Let us analyze the minimal $U(1)_{xq-\tau_R^3}$ model with $x_{\min} = 25/123$, where the value of K is minimized. The $U(1)_{xq-\tau_R^3}$ charge for H causes $\lambda_{\text{mix}} \neq 0$ in the IR scale. This is potentially dangerous, because the ρ -parameter deviates from unity at the tree level. We also note that in the minimal $U(1)_{xq-\tau_R^3}$ model, the β function for the gauge mixing term g_{mix} is vanishing. Because the charge of $U(1)_{xq-\tau_R^3}$ is given by $Q = 6xY - \tau_R^3$, we find the derivative of Eq. (6) with respect to x as

$$\frac{\partial a}{\partial x} = \frac{4}{3} \sum_f Q_f \frac{\partial Q_f}{\partial x} + \frac{2}{3} \sum_s Q_s \frac{\partial Q_s}{\partial x} = 12 \left[\frac{2}{3} \sum_f Q_f Y_f + \frac{1}{3} \sum_s Q_s Y_s \right] = 12 a_{\text{mix}}, \quad (38)$$

where a_{mix} is the coefficient of the β function for g_{mix} , which is not proportional to g_{mix} (see Appendix A). If we take $x = x_{\min}$, $\partial a / \partial x = 0$ holds and $a_{\text{mix}} = 0$ is satisfied. Then if we set the gauge mixing term as $g_{\text{mix}}(\Lambda) = 0$ at the UV scale Λ , it continues to be zero at any energy scale; $g_{\text{mix}}(\mu) \equiv 0$. This property reduces the labor of the numerical analysis.

We show the values of $\delta\rho$ in Fig. 1. Allowing 1σ deviation, we find the lower bound for the Z' mass as $M_{Z'} \gtrsim 0.8$ TeV. The results are almost independent of the number of ν_R having the relevant Majorana Yukawa couplings.

We also depict the values of $\alpha_{B-L} = g_{B-L}^2/(4\pi)$ at $\mu = v_\Phi$ in Fig. 2. We here used the notation g_{B-L} instead of $g_{1'}$ in order to compare the results with the previous ones [8–10, 13]. In comparison with the pure $B-L$ model shown in Fig. 3 in Ref. [13], the obtained gauge coupling is so small and thereby $\delta\rho$ is reduced from the naively expected one.

We further show the extra Higgs mass m_ϕ in Fig. 3. Although the values of m_ϕ depend on N_ν , we find roughly $m_\phi \sim \mathcal{O}(\text{GeV})$. Compared with the pure $B-L$ model shown in Fig. 2 in Ref. [13], m_ϕ is also suppressed because of the very tiny β function $|\beta_{\lambda_\Phi}| \ll 1$. The mass of the right-handed neutrinos M_{ν_R} can be easily estimated from the CW relation (36), $M_{\nu_R} \simeq \sqrt[4]{\frac{3}{2N_\nu}} M_{Z'}$, i.e., the left-hand side of (36) is saturated by M_{ν_R} .

B. Minimal vector-like $U(1)'$

Let us analyze the minimal vector-like $U(1)'$ model. For simplicity, we take $N_g = 3$, $N_\Phi = N_\psi = N_{\nu_R} = N_{N_R} = 1$, and $x = y = 1$.

³ This is consistent with the indirect prediction, $\overline{m}_t = 167.5_{-7.3}^{+8.9}$ GeV [20], while the converted value to the pole mass [26] is rather small, compared to the directly obtained value at the Tevatron/LHC.

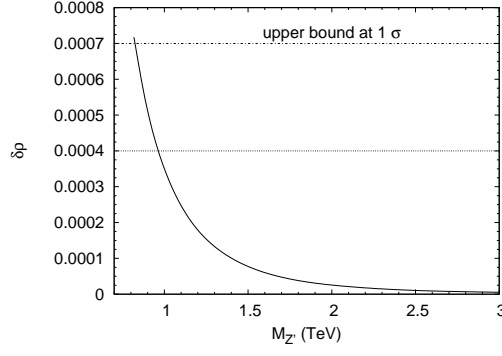


FIG. 1: $\delta\rho$ for the minimal $U(1)_{xq-\tau_R^3}$ model with $x = 25/123$. The dotted and dash-dotted lines correspond to the central value and the upper bound at 1σ , respectively. The results are almost the same for $N_\nu = 1, 2, 3$, where N_ν represents the number of the right-handed neutrinos having the relevant Majorana Yukawa couplings.

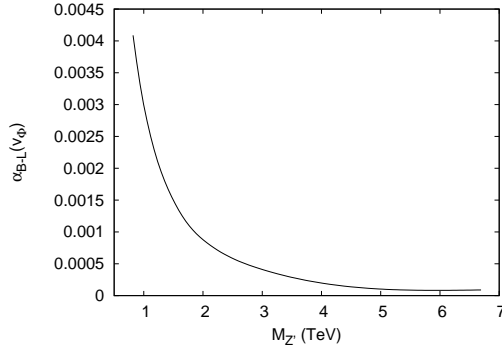


FIG. 2: $M_{Z'}$ v.s. $\alpha_{B-L}(v_\Phi)$ for the minimal $U(1)_{xq-\tau_R^3}$ model with $x = 25/123$. The results are almost the same for $N_\nu = 1, 2, 3$.

We depict the results for $\alpha_{1'} = g_{1'}^2/(4\pi)$ at $\mu = v_\Phi$ in Fig. 4. The values and behaviors are similar to those in the pure $B-L$ model shown in Fig. 3 in Ref. [13]. In this model, the deviation of the ρ -parameter comes only from $g_{\text{mix}} \neq 0$ in low energy, so that it is tiny, $\delta\rho \lesssim 10^{-6}$.

We also show the extra Higgs mass m_ϕ in Fig. 5. The values of m_ϕ are, say, $m_\phi \sim \mathcal{O}(100 \text{ GeV})$, and much larger than those of $U(1)_{xq-\tau_R^3}$. This is because K is closer to one, and consequently $|\beta_{\lambda_\Phi}| \ll 1$, in the minimal $U(1)_{xq-\tau_R^3}$ model.

V. SUMMARY

We investigated several models with a $U(1)$ extension and checked whether the condition $K < 1$ for the flatland scenario is satisfied. As shown in Ref. [13], the pure $B-L$ models with $N_g = 1, 2$ and $N_\nu = 1$ satisfy $K < 1$, while the familiar $B-L$ model with $N_g = 3$ does not. We found that twisted versions of the $B-L$ model, $U(1)_{xq-\tau_R^3}$, and $U(1)_{xd-u+yY}$ models satisfy the condition even for $N_g = 3$. We also proposed a minimal vector-like $U(1)'$ model with $K < 1$.

In particular we explicitly calculated the behavior of the running coupling constants in the minimal $U(1)_{xq-\tau_R^3}$ model and in the minimal vector-like $U(1)'$ model, and confirmed the realizations of the flatland scenario. Although the ρ -parameter deviates from unity at the tree level in the $U(1)_{xq-\tau_R^3}$ model, the deviation is small for $M_{Z'} \gtrsim 0.8 \text{ TeV}$ because of the very small gauge coupling. For the minimal vector-like $U(1)'$ model, since the deviation $\delta\rho \neq 0$ appears essentially from the one-loop corrections, it is suppressed as $\delta\rho \lesssim 10^{-6}$; the model is not strongly constrained by the ρ -parameter. The setup of the minimal vector-like $U(1)'$ model is similar to the semi-invisible Z' model [16]. The Z_2 -odd fermion N_R in the model can be a candidate of the dark matter. Also, the Z - Z' mixing effect can be explored

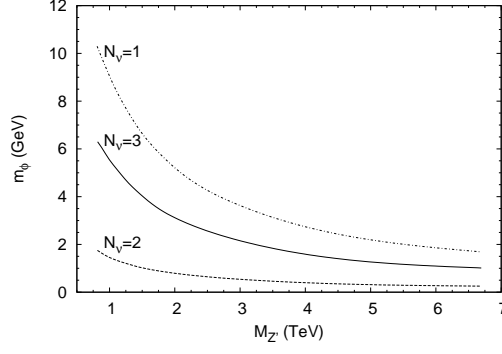


FIG. 3: $M_{Z'}$ v.s. m_ϕ for the minimal $U(1)_{xq-\tau_R^3}$ model with $x = 25/123$. N_ν denotes the number of the right-handed neutrinos having the relevant Majorana Yukawa couplings.

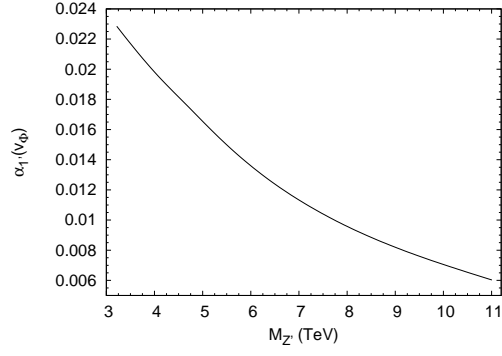


FIG. 4: $M_{Z'}$ v.s. $\alpha_1(v_\Phi)$ for the minimal vector-like $U(1)'$ model. We took $N_\Phi = N_\psi = N_{\nu_R} = N_{N_R} = 1$ and $x = y = 1$.

at the LHC and/or at the future ILC [24, 25]. Further investigations will be performed elsewhere.

Appendix A: RGEs for $U(1)_{xq-\tau_R^3}$

We show the full set of the RGEs for the $U(1)_{xq-\tau_R^3}$ model.

The RGEs for the SM gauge couplings are

$$(16\pi^2)\mu \frac{\partial}{\partial \mu} g_i = c_i g_i^3, \quad (A1)$$

with

$$c_Y = \frac{41}{6}, \quad c_2 = -\frac{19}{6}, \quad c_3 = -7, \quad (A2)$$

The RGEs for $U(1)'$ are

$$(16\pi^2)\mu \frac{\partial}{\partial \mu} g_{B-L} = g_{B-L} \left[a g_{B-L}^2 + 2a_{\text{mix}} g_{B-L} g_{\text{mix}} + c_Y g_{\text{mix}}^2 \right], \quad (A3)$$

$$(16\pi^2)\mu \frac{\partial}{\partial \mu} g_{\text{mix}} = g_{\text{mix}} \left[c_Y (g_{\text{mix}}^2 + 2g_Y^2) + a g_{B-L}^2 \right] + 2a_{\text{mix}} g_{B-L} (g_{\text{mix}}^2 + g_Y^2), \quad (A4)$$

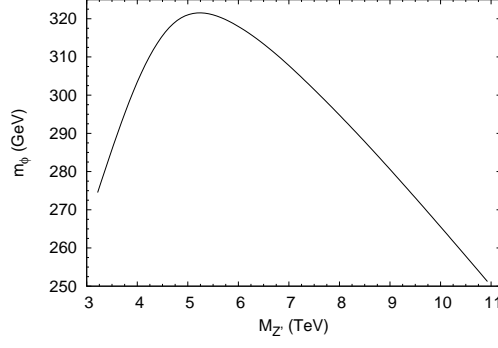


FIG. 5: $M_{Z'}$ v.s. m_ϕ for the minimal vector-like $U(1)'$ model. We took $N_\Phi = N_\psi = N_{\nu_R} = N_{N_R} = 1$ and $x = y = 1$.

with

$$a = \left(80x^2 - 32x + \frac{16}{3}\right) N_g + \frac{4}{3} N_\Phi + \frac{2}{3} (3x-1)^2 N_H, \quad (\text{A5})$$

$$a_{\text{mix}} = \left(\frac{40}{3}x - \frac{8}{3}\right) N_g + \left(x - \frac{1}{3}\right) N_H. \quad (\text{A6})$$

For the Yukawa couplings, we find

$$(16\pi^2)\mu \frac{\partial}{\partial \mu} y_t = y_t \left[\frac{9}{2} y_t^2 - \left(8g_3^2 + \frac{9}{4} g_2^2 + \frac{17}{12} (g_Y^2 + g_{\text{mix}}^2) + 3(17x^2 - 8x + 1)g_{B-L}^2 + (17x - 4)g_{B-L} g_{\text{mix}} \right) \right], \quad (\text{A7})$$

and

$$(16\pi^2)\mu \frac{\partial}{\partial \mu} y_M = y_M \left[(4 + 2N_\nu) y_M^2 - 6g_{B-L}^2 \right], \quad (\text{A8})$$

where $Y_M^{ij} = \text{diag}(y_M, \dots, y_M, 0, \dots, 0)$, $\text{tr}[(Y_M^{ij})^2] = N_\nu y_M^2$, etc. with N_ν being the number of the large Majorana couplings. The RGEs for the Higgs quartic couplings are

$$(16\pi^2)\mu \frac{\partial \lambda_H}{\partial \mu} = 24\lambda_H^2 + \lambda_{\text{mix}}^2 + \frac{3}{8} \left(2g_2^4 + \{g_2^2 + g_Y^2 + (g_{\text{mix}} + 2(3x-1)g_{B-L})^2\}^2 \right) - 3\lambda_H \{3g_2^2 + g_Y^2 + (g_{\text{mix}} + 2(3x-1)g_{B-L})^2\} - 6y_t^4 + 12\lambda_H y_t^2, \quad (\text{A9})$$

$$(16\pi^2)\mu \frac{\partial \lambda_\Phi}{\partial \mu} = 20\lambda_\Phi^2 + 2\lambda_{\text{mix}}^2 - 16N_\nu y_M^4 + 8N_\nu \lambda_\Phi y_M^2 + 96g_{B-L}^4 - 48\lambda_\Phi g_{B-L}^2, \quad (\text{A10})$$

$$(8\pi^2)\mu \frac{\partial \lambda_{\text{mix}}}{\partial \mu} = \lambda_{\text{mix}} \left[6\lambda_H + 4\lambda_\Phi + 2\lambda_{\text{mix}} + 3y_t^2 + 2N_\nu y_M^2 - \frac{3}{4} \{3g_2^2 + g_Y^2 + (g_{\text{mix}} + 2(3x-1)g_{B-L})^2\} - 12g_{B-L}^2 \right] + 6(g_{\text{mix}} + 2(3x-1)g_{B-L})^2 g_{B-L}^2. \quad (\text{A11})$$

The minimal $U(1)_{xq-\tau_R^3}$ corresponds to $x = x_{\text{min}} = \frac{25}{123}$, which yields $\min(a) = \frac{964}{123}$ and minimizes K . In this case, we also find $a_{\text{mix}} = 0$, so that $g_{\text{mix}}(\mu) \equiv 0$ in all energy scale. This choice is allowed, because $\lambda_{\text{mix}}(\Lambda) = 0$ does not mean $\lambda_{\text{mix}}(\mu) \equiv 0$ at any μ , owing to the g_{B-L}^4 term in $\frac{\partial \lambda_{\text{mix}}}{\partial \ln \mu}$. Even in this case, the values of $\delta\rho$ are kept within the experimental bounds, if $M_{Z'} \gtrsim 0.8$ TeV.

Appendix B: RGEs for the minimal vector-like $U(1)'$ model

We show the full set of the RGEs for the minimal vector-like $U(1)'$ model.

The RGEs for the SM gauge couplings are

$$(16\pi^2)\mu \frac{\partial}{\partial \mu} g_i = c_i g_i^3, \quad (\text{B1})$$

with

$$c_Y = \frac{41}{6} + \frac{4x^2}{3}N_\psi, \quad c_2 = -\frac{19}{6}, \quad c_3 = -7, \quad (\text{B2})$$

The RGEs of $g_{1'}$ and g_{mix} are

$$(16\pi^2)\mu \frac{\partial}{\partial \mu} g_{1'} = g_{1'} \left[a g_{1'}^2 + 2a_{\text{mix}} g_{1'} g_{\text{mix}} + c_Y g_{\text{mix}}^2 \right], \quad (\text{B3})$$

$$(16\pi^2)\mu \frac{\partial}{\partial \mu} g_{\text{mix}} = g_{\text{mix}} \left[c_Y (g_{\text{mix}}^2 + 2g_Y^2) + a g_{1'}^2 \right] + 2a_{\text{mix}} g_{1'} (g_{\text{mix}}^2 + g_Y^2), \quad (\text{B4})$$

with

$$a = \frac{4}{3}N_g + \frac{4}{3}N_\Phi + \frac{4y^2}{3}N_\psi, \quad a_{\text{mix}} = \frac{4xy}{3}N_\psi. \quad (\text{B5})$$

The RGE for the top Yukawa coupling is

$$(16\pi^2)\mu \frac{\partial}{\partial \mu} y_t = y_t \left[\frac{9}{2}y_t^2 - \left(8g_3^2 + \frac{9}{4}g_2^2 + \frac{17}{12}(g_Y^2 + g_{\text{mix}}^2) \right) \right], \quad (\text{B6})$$

and also those for the Majorana Yukawa couplings are

$$(16\pi^2)\mu \frac{\partial}{\partial \mu} y_M = y_M \left[(4 + 2N_{\nu_R})y_M^2 + 2N_{N_R}y_N^2 - 6g_{1'}^2 \right], \quad (\text{B7})$$

and

$$(16\pi^2)\mu \frac{\partial}{\partial \mu} y_N = y_N \left[(4 + 2N_{N_R})y_N^2 + 2N_{\nu_R}y_M^2 - 6g_{1'}^2 \right], \quad (\text{B8})$$

where we took $Y_{M(N)}^{ij} = \text{diag}(y_{M(N)}, \dots, 0, \dots)$, $\text{tr}[(Y_{M(N)}^{ij})^2] = N_{\nu_R(N_R)}y_{M(N)}^2$, etc. with $N_{\nu_R(N_R)}$ being the number of the large Majorana couplings.

The RGEs for the scalar quartic couplings are

$$(16\pi^2)\mu \frac{\partial \lambda_H}{\partial \mu} = 24\lambda_H^2 + \lambda_{\text{mix}}^2 + \frac{3}{8} \left(2g_2^4 + (g_2^2 + g_Y^2 + g_{\text{mix}}^2)^2 \right) - 6y_t^4 + 12\lambda_H y_t^2 - 3\lambda_H (3g_2^2 + g_Y^2 + g_{\text{mix}}^2), \quad (\text{B9})$$

$$(16\pi^2)\mu \frac{\partial \lambda_\Phi}{\partial \mu} = 20\lambda_\Phi^2 + 2\lambda_{\text{mix}}^2 - 16(N_{\nu_R}y_M^4 + N_{N_R}y_N^4) + 8(N_{\nu_R}y_M^2 + N_{N_R}y_N^2)\lambda_\Phi + 96g_{1'}^4 - 48\lambda_\Phi g_{1'}^2, \quad (\text{B10})$$

$$(8\pi^2)\mu \frac{\partial \lambda_{\text{mix}}}{\partial \mu} = \lambda_{\text{mix}} \left[6\lambda_H + 4\lambda_\Phi + 2\lambda_{\text{mix}} + 3y_t^2 + 2N_{\nu_R}y_M^2 + 2N_{N_R}y_N^2 - \frac{3}{4}(3g_2^2 + g_Y^2 + g_{\text{mix}}^2) - 12g_{1'}^2 \right] + 6g_{1'}^2 g_{\text{mix}}^2. \quad (\text{B11})$$

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